Reconstructing Network Structures from Partial Measurements

Melvyn Tyloo

UNIVERSITÉ DE GENÈVE
FACULTÉ DES SCIENCES

Swiss National Science Foundation

Dynamics Days Europe 2021, Nice, France.

website: melvyntyloo.com

melvyn.tyloo@gmail.com

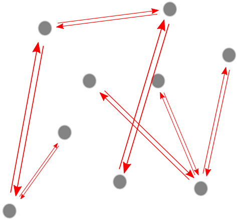
MT, R. Delabays, P. Jacquod, researchsquare (2021)

Complex Coupled System

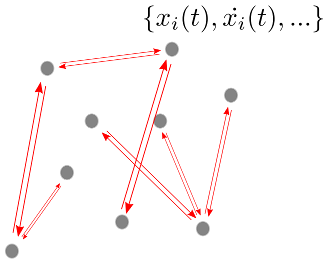
$$\{x_i(t), \dot{x}_i(t), ...\}$$

Complex Coupled System

$\{x_i(t), \dot{x}_i(t), ...\}$



Complex Coupled System



Examples:

Power grid
$x_i(t) \rightarrow$ voltage angle
$\dot{x}_i(t) \rightarrow$ voltage frequency

Social dynamics
$x_i(t) \rightarrow$ agent opinion

Finance
$x_i(t) \rightarrow$ stock or option price
...

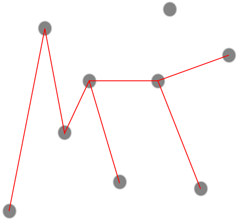
Network Characteristics

Number of nodes N?

Network connectivity?

Partial network structures?

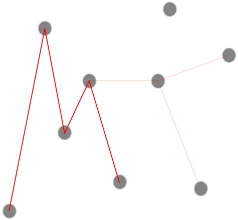
Network Characteristics



Number of nodes N?

Network connectivity?

Partial network structures?

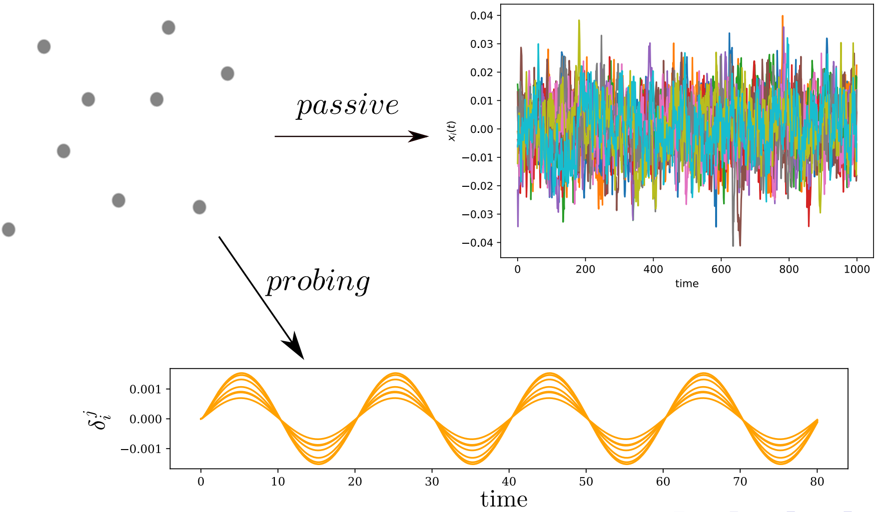


Number of nodes N?

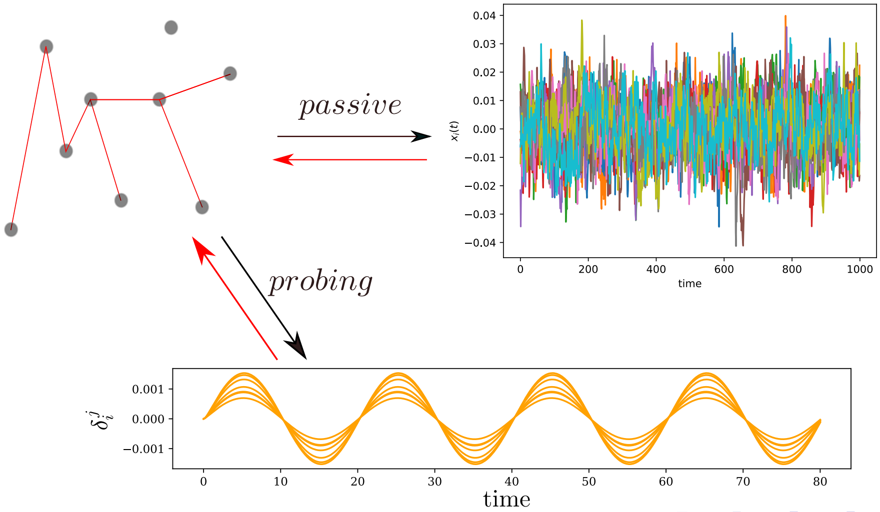
Network connectivity?

Partial network structures?

Times-Series from a priori coupled system



Times-Series from a priori coupled system



Mathematical Model

Diffusively Coupled Agents

$$\dot{x}_i(t) = \omega_i - \sum_j a_{ij} f_{ij}(x_i - x_j) + \xi_i(t) + b_i(t) \ , i = 1, ..., n \,. \qquad (1)$$

a_{ij} adjacency matrix elements.

$$a_{ij} \begin{cases} > 0, & \text{if } i,j \text{ are connected,} \\ 0, & \text{otherwise.} \end{cases} \qquad (2)$$

ξ_i noise.
b_i probing/input signal.

Diffusively Coupled Agents

$$\dot{x}_i(t) = \omega_i - \sum_j a_{ij} f_{ij}(x_i - x_j) + \xi_i(t) + b_i(t)\,,\, i = 1, ..., n\,. \tag{3}$$

a_{ij} adjacency matrix elements.

$$a_{ij} \begin{cases} > 0, & \text{if } i, j \text{ are connected,} \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

ξ_i noise.
b_i probing/input signal.

$$\{x_i(t), \dot{x}_i(t)\} \rightarrow a_{ij},\ n\,.$$

Mathematical Model

Diffusively Coupled Agents

$$\dot{x}_i(t) = \omega_i - \sum_j a_{ij} f_{ij}(x_i - x_j) + \xi_i(t) + b_i(t)\ , i = 1, ..., n\,. \tag{5}$$

a_{ij} adjacency matrix elements.

$$a_{ij} \begin{cases} > 0, & \text{if } i, j \text{ are connected,} \\ 0, & \text{otherwise.} \end{cases} \tag{6}$$

ξ_i noise.
b_i probing/input signal.

Assumption

- system stays close to a stable fixed point x^*.

Assumption

- system stays close to a stable fixed point x^*.

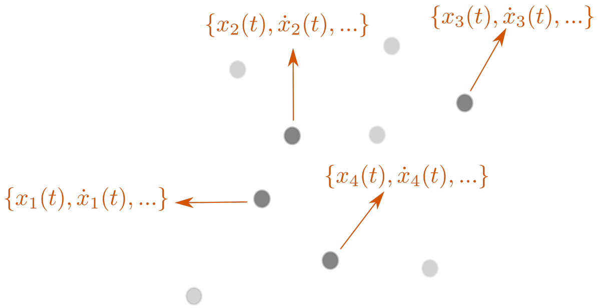
Linearization

$$\delta\dot{x}_i(t) = \omega_i - \underbrace{\sum_j a_{ij}\frac{\partial f_{ij}}{\partial x}(x^*)(\delta x_i - \delta x_j)}_{\sum_j \mathbb{J}_{ij}\delta x_j} + \xi_i(t) + b_i(t)\ , i = 1, ..., n\,. \quad (7)$$

$$\mathbb{J}_{ij} = \begin{cases} -a_{ij}\frac{\partial f_{ij}}{\partial x}(x^*), & \text{if } i \neq j, \\ \sum_k a_{ik}\frac{\partial f_{ik}}{\partial x}(x^*), & \text{if } i = j. \end{cases} \quad (8)$$

Passive observation

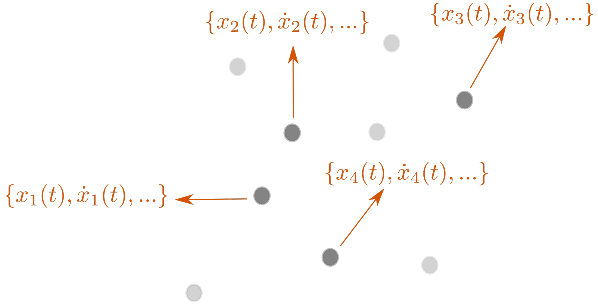
$$\delta\dot{x}_i(t) = \omega_i - \sum_j \mathbb{J}_{ij}\delta x_j + \xi_i(t)\ , i = 1, ..., n\,. \quad (9)$$



Network Connectivity

Passive observation

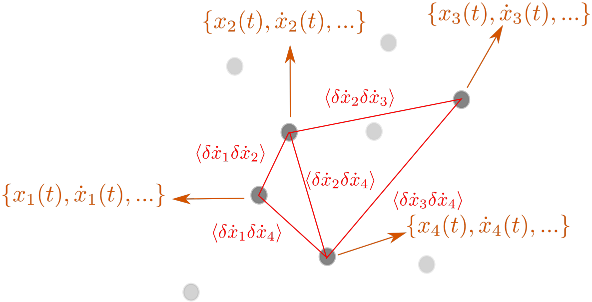
$$\delta\dot{x}_i(t) = \omega_i - \sum_j \mathbb{J}_{ij}\delta x_j + \xi_i(t)\ , i = 1, ..., n\,. \tag{10}$$



- If access to every nodes $\rightarrow \langle \delta x_i \delta x_j \rangle \propto \mathbb{J}^\dagger$ (Ren et al. PRL 2010).

Passive observation

$$\delta\dot{x}_i(t) = \omega_i - \sum_j \mathbb{J}_{ij}\delta x_j + \xi_i(t)\,,\, i = 1, ..., n\,. \tag{11}$$



- If access to every nodes $\rightarrow \langle\delta x_i \delta x_j\rangle \propto \mathbb{J}^\dagger$ (Ren et al. PRL 2010).
- If access only to a subset of nodes $\rightarrow \langle\delta\dot{x}_i \delta\dot{x}_j\rangle \propto \mathbb{J}$ (MT, Delabays, Jacquod 2021).

Noise vs. System Time-scales

- Time-correlated noise

$$\begin{aligned} \langle \xi_i(t) \rangle &= 0\,, \\ \langle \xi_i(t+\Delta t/2)\xi_j(t-\Delta t/2) \rangle &= \xi_0^2\,\delta_{ij}\exp\left(-|\Delta t|/\tau_0\right)\,, \end{aligned} \tag{12}$$

correlation time τ_0 .

- Two-point correlator

$$\lim_{t\to\infty}\langle \delta\dot{x}_i(t)\delta\dot{x}_j(t) \rangle = \xi_0^2\left(\delta_{ij} - \sum_\alpha u_{\alpha,i}u_{\alpha,j}\frac{\lambda_\alpha\tau_0}{1+\lambda_\alpha\tau_0}\right), \tag{13}$$

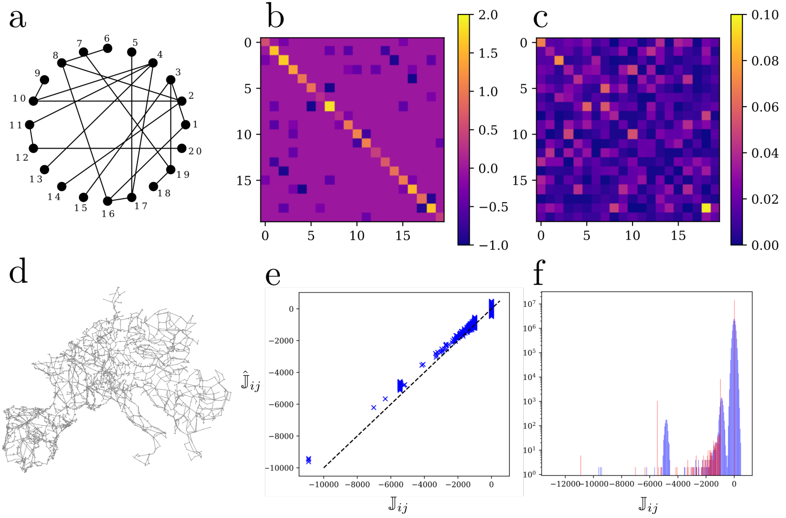
Lyapunov exponents λ_α .

Noise vs. System Time-scales

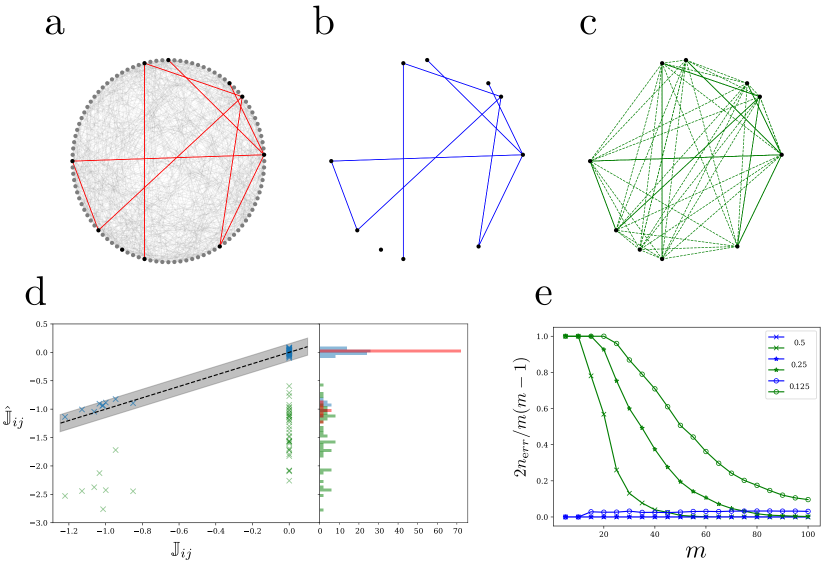
White noise limit: $\lambda_\alpha \tau_0 \ll 1$,

$$\hat{\mathbb{J}}_{ij} = \left(\delta_{ij} - \langle \delta\dot{x}_i \delta\dot{x}_j \rangle / \xi_0^2\right) \tau_0^{-1} \,. \tag{14}$$

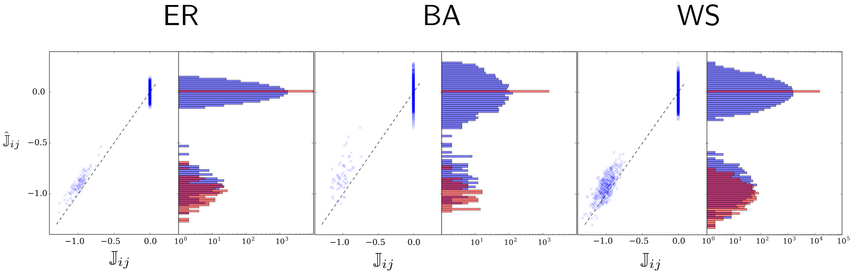
Network Connectivity



Network Connectivity

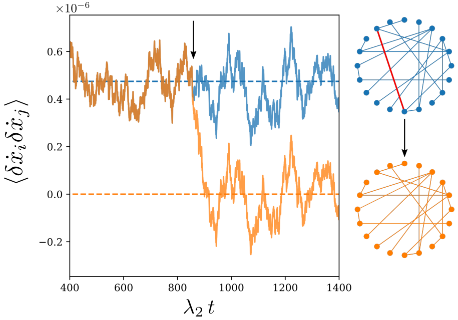


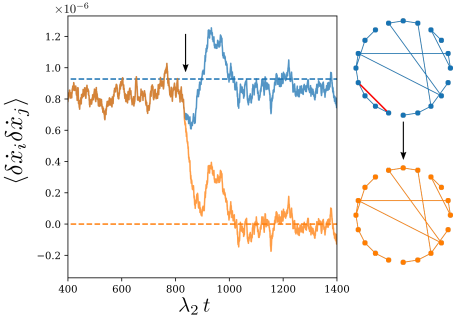
Large Networks



$n = 1000\,,\ m \cong 100\,.$

Real-time detection



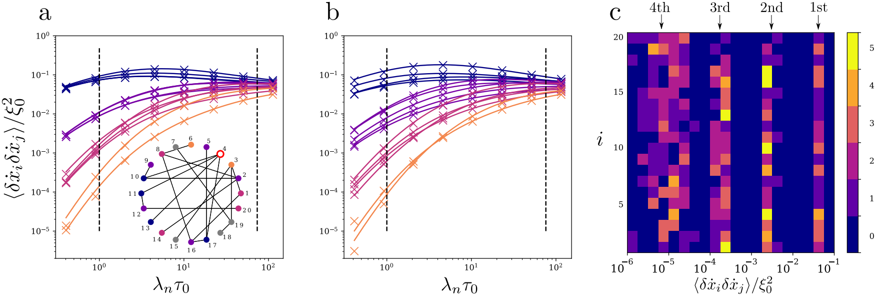


Geodesic distances

$$\langle \delta\dot{x}_i \delta\dot{x}_j \rangle = \xi_0^2 \sum_{k=q}^{\infty} (-\tau_0)^k \left(\mathbb{J}^k\right)_{ij} \,. \tag{15}$$

$$\mathrm{min}_{l,m}(\mathbb{J}^{q-1})_{lm}\tau_0^{-1} \gg (\mathbb{J}^q)_{ij} \gg \mathrm{max}_{l,m}(\mathbb{J}^{q+1})_{lm}\tau_0 \,, \tag{16}$$

Geodesic distances



Conclusion

Network inference

- Reconstruct network within the accessible units.
- Real-time detection of changes in the topology.
- Geodesic distances.

MT, R. Delabays, P. Jacquod, researchsquare (2021)

Other Questions of Interest

- Probing
- Fault location
- Fault identification
- Prediction of extreme events
- ...

R. Delabays, MT, IFAC-PapersOnLine **54**(9), 696-700 (2021)
MT, R. Delabays J. Phys. Complex. **2** 025016 (2021)
R. Delabays, L. Pagnier, MT New J. Phys. **23** 043037 (2021)

CCS 2021: Satellite

CCS2021 Satellite Symposium

Data-based Diagnosis of Networked Dynamical Systems
Wednesday October 27th, 2021

Confirmed speakers

- Misha Chertkov, *University of Arizona, USA.*
- Pietro De Lellis, *University of Naples Federico II, Italy.*
- Philippe Jacquod, *University of Geneva, Switzerland.*
- Nathan Kutz, *University of Washington, USA.*
- Andrey Lokhov, *Los Alamos National Laboratory, USA.*
- Enrique Mallada, *Johns Hopkins University, USA.*
- Edward Ott, *University of Maryland, USA.*
- Tiago Peixoto, *Central European University, Austria-Hungary.*
- Leonardo Rydin Gorjão, *Forschungszentrum Jülich, Germany.*
- Marc Timme, *Technische Universität Dresden, Germany.*
- Marc Vuffray, *Los Alamos National Laboratory, USA.*
- Gil Zussman, *Columbia University, USA.*

https://www.delabaysrobin.site/ccs-satellite